

# METHOD OF ANALYSIS AND SYNTHESIS IN EDUCATION OF MATHEMATICS

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The method of analysis and synthesis has long history which started from Ancient geometry and it is “a method Greek geometers used in looking for proofs of theorems (theoretical analysis) and for constructions to solve problems (problematical analysis)” (Hintikka & Remes 1974, 1). It is not clear how to specify the method, but generally it is accepted that the analysis starts from what is eventually sought for and that it is followed by synthesis which is formulated in strict logical form. Usually textbooks follow their own logical form which is, of course, part and parcel of mathematical practise. However, the approach hides the constructive aspects of geometrical and, more generally, mathematical reasoning (Mumma 2012).

More technically, the logical form adduces “the analysis-of-proof view” (Hintikka & Remes 1974, 31) which analyses the steps of deduction. Unfortunately, this does not explicate the essential constructive aspects of mathematical and geometrical reasoning. The idea of analysis is to proceed “as an ‘upward’ movement, that is, as a search of premises rather than as a sequence of conclusions” (Hintikka & Remes 1974, xiv) which is expressed by “the analysis-of-figures view” (Hintikka & Remes 1974, 32). The idea is to search for the auxiliary constructions needed in the formal proofs which makes the analysis constructive (Hintikka & Remes 1974; Niiniluoto 2018).

The idea of constructions is to bring new information into the reasoning process. The additional information can be expressed in linguistic form which takes place especially in algebraic contexts or in pictorial form which takes place especially in geometry which emphasizes visual expressions in reasoning (Priest, De Toffoli & Findlen 2018, 49). At the same, this shows why the logical way to express mathematics entails that “visual representation remains a second-class citizen in both the theory and practice of mathematics” (Barwise & Etchemendy 1991, 9). Moreover, the systematic use of the method of analysis and synthesis shows the methodological role of mathematics in different fields of science: mathematics is not merely a formal tool but, at the same, methodological model of constructive knowledge acquisition. (Hintikka & Remes 1974; Priest, De Toffoli & Findlen 2018; Niiniluoto 2018).

The role of reasoning in mathematics education needs further research since constructions are important factors for mathematics learning. In the presentation, methodology issues will be connected to the methodology of analysis and synthesis.

## References

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